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ELECTRICAL ANALOGS OF STATICALLY LOADED STRUCTURES

by Frederick L. Ryder

ENGINEERING MECHANICS DIVISION

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PAPERS

ELECTRICAL ANALOGS OF STATICALLY LOADED STRUCTURES

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Synopsis

The labor involved in the solution of indeterminate structures may be greatly reduced by the use of electrical analogs, especially in those cases where various choices of dimensions are to be investigated. There is a similarity between the structure and its analog which often permits the construction of the analog on a pictorial basis without regard to the governing equations.

In the electrical circuit, force and moment are simulated by current, and deflection and slope by voltage drop. The analogy is based on the fact that Castigliano's theorem and the principle of least work have their counterparts in certain electrical networks. The theory is developed for the two-dimensional and three-dimensional truss, and for two-dimensional rigid frames and truss-frame combinations, including trusses with secondary stresses.

A working model of the necessary equipment, consisting of adjustable resistors and tapped transformers is described, and instructions are given for its use. Experimental results that are accurate to within 2.5% of the maximum values encountered have been obtained. A rapid numerical check is possible.

Introduction

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically for convenience of reference in the Appendix.

Determinate trusses and rigid frames are relatively simple to analyze, since all forces and moments can be evaluated by considerations of equilibrium. The solution of indeterminate structures is contrastingly tedious because the forces and moments depend on the mutual compatibility of the deflections and slopes, which are functions of the elastic parameters, as well as on equilibrium considerations.

An electrical instrument for the experimental solution of indeterminate

framed structures had been devised by Vannevar Bush.[2] The instrument consists of adjustable resistors and tapped transformers which can be connected to simulate trusses and rigid frames of different configurations—the properties of the individual members being simulated by suitable setting of the resistors and selection of the transformer taps. Currents are introduced to simulate applied loads, and the internal forces and moments are simulated by certain other currents which are easily measured. This type of device has the advantage over purely mathematical instruments in that the governing equations need not be derived, and is superior to mechanical models in regard to the speed and economy of changing the dimensions of the structure under investigation.

[2] "Structural Analysis by Electric Circuit Analogies," by Vannevar Bush, *Journal*, The Franklin Institute, Vol. 217, March, 1934, p. 289.

The present paper continues Mr. Bush's work in the respect that for trusses, the analysis of the analogous relationships existing between the structure and its corresponding electrical circuit are extended and generalized. For rigid frames, Mr. Bush introduced an analog for the two-dimensional case, in which the members were straight and disposed rectangularly. The analogy is based on the method of slope deflection, and requires the explicit simulation of the slopes and deflections as well as the moments and shears. This paper describes an analog which is applicable to all two-dimensional rigid frames, including those with curved members. Since this analog is based on the principle of least work, slope and deflection are not represented explicitly, the simulation being conducted in terms of forces, moments, and the elastic properties of the members. This analog can be extended to the three-dimensional rigid frame and to nonlinear rigid frames and trusses.

An approach quite different from Mr. Bush's has been followed by Gabriel Kron, who has described an electrical analog applicable to any framed structure in which the members are straight.[3] Mr. Kron's circuit elements consist of adjustable condensers and inductances, rather than the resistors and transformers used by Mr. Bush and the writer.

[3] "Tensor Analysis and Equivalent Circuits of Elastic Structures," by Gabriel Kron, *Journal*, The Franklin Institute, Vol. 238, December, 1944, p. 399.

Analogous Structural–Electrical Energy Theorems

The theorems on which the analogs are based are herewith summarized.[4]

[4] "Network Analysis by Least Power Theorems," by F. L. Ryder, *Journal*, The Franklin Institute, Vol. 254, July, 1952, p. 47.

Principle of Least Power.—In a circuit of resistors and ideal transformers that is supplied with given external alternating currents at a single frequency and phase angle, each internal current is a minimum subject to the requirements of current-continuity. This is analogous to the principle of least work, which states that each internal force and moment in a structure is a minimum, subject to the requirements of equilibrium.

Castigliano's Theorem for Electrical Circuits.—In the circuit previously described, P denotes the power dissipated in the circuit; C is a supplied current, considered positive when leaving the circuit; and V denotes the voltage (measured below some arbitrary datum or ground) of the point at which C leaves the circuit.

It can be shown that

$$V = \tfrac{1}{2} \frac{\partial P}{\partial C} \quad \ldots\ldots (1)$$

Eq. 1 is analogous to Castigliano's theorem for structures, which states that

$$\delta = \frac{\partial W_d}{\partial F} \quad \ldots\ldots (2)$$

and

$$\theta = \frac{\partial W_d}{\partial M} \quad \ldots\ldots (3)$$

in which W_d is the work of deformation; F and M are applied force and bending moment, respectively; and δ and θ are the deflection and slope, respectively, measured at the point of application of F or M. The absence of the factor $\frac{1}{2}$ in Eqs. 2 and 3 arises from the fact that electrical power equals the product of current and voltage, whereas the work of deformation is equal to one half the product of force and deflection, or of moment and slope. The following parameters may therefore be considered analogous:

Structure	Electrical circuit
Force	Current
Moment	Current
Deflection	Voltage
Slope	Voltage
Work	One half the power

The Ideal Transformer.—In constructing analogs based on the foregoing principles, ideal transformers are used. The behavior of such transformers is specified by the follow rules:

(1) The ratio of secondary voltage drop to primary voltage drop equals the turns ratio N.

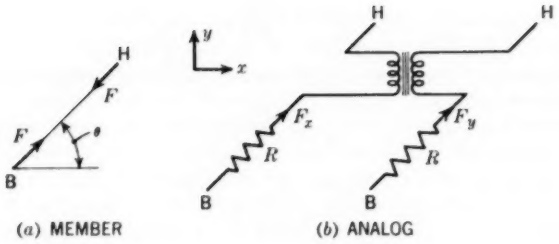
(2) The ratio of secondary current to primary current is the negative reciprocal of the turns ratio N.

The voltage drop and the current must be measured in a selected direction for each winding. The number of turns is positive if positive current produces a flux in a specified direction in the transformer core; otherwise it is negative. These rules result in zero power loss in the transformer. The errors caused by the difference between practical and ideal transformers will be discussed subsequently.

Analog of the Pin-Connected Truss

Two-Dimensional Case.—A typical two-dimensional truss member, BH, is shown in Fig. 1(*a*). Its analog consists of an x-branch and a y-branch, connected by an ideal transformer as shown in Fig. 1(*b*). The arbitrarily oriented Cartesian axes are indicated, as well as the angle of inclination, θ, the latter being measured from the x-axis to the direction of member BH. The reference direction for θ is that in which the y-axis is rotated 90° from the x-axis.

The force in branch BH is designated as F, and is considered positive if



(*a*) MEMBER (*b*) ANALOG

FIG. 1.—TWO-DIMENSIONAL TRUSS MEMBER

tensile. The alternating currents in the electrical branches are designated as F_x and F_y, respectively, and are at the same frequency and phase—the transformer turns ratio being such that these currents are in the same ratio as the x- and y-components of force F, measured in the branch direction chosen for the currents. To achieve this, the following must be satisfied:

$$\frac{N_x}{N_y} = -\tan\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which N_x and N_y are the number of turns in the x- and y-windings of the transformer, respectively. The resultant current, whose rectangular components are F_x and F_y, is designated as F, since it is analogous to branch force. This resultant is not explicit in the analog.

The analog of the entire truss of which branch BH is a member consists of a separate x- and y-network, each consisting of one branch for each truss member, the branches being connected in the same way as are the truss members. The separate networks are interconnected only magnetically—through the transformer cores. The x- and y-components of the external forces, including both loads and reactions, are simulated by appropriately supplied x- and y-currents, at the same frequency and phase, the reference direction for these currents being considered as positive when leaving the network. According to the principles of least work and least power, the truss will be properly simulated if the equations of equilibrium are duplicated by the equations of current-continunity, and twice the work of deformation is duplicated by the power dissipation.

To compare the equations of equilibrium and current-continuity, the sum of the forces applied to joint B in the truss, and the currents leaving joint B in the analog are equated to zero. In the former case,

$$F'_x + F\cos\theta = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

and

$$F'_y + F\sin\theta = 0, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5b)$$

in which F'_x and F'_y are, respectively, the x- and y-components of the resultant of all the forces applied to joint B by branches other than member BH and by the external forces, which include both loads and reactions. In the electrical

case, Eq. 5 will be satisfied if F'_x is the total current, other than F_x, leaving joint B in the x-network, and can be composed of branch currents and supplied currents. A similar result is obtained for F'_y.

The force and current summations at joint H are similar to those at joint B, except that all terms are prefixed by a minus sign. Hence, if force components applied to a truss joint are represented by currents leaving the corresponding electrical joints, then current is analogous to force, in so far as the equations of equilibrium and current-continuity are concerned.

The work of deformation of each branch is $\frac{F^2 L}{2 A E}$, in which L is the length, A denotes the cross-sectional area, and E is the modulus of elasticity. The electrical power equals twice the work of deformation, a result that can be achieved if the branch resistance R in Fig. 1 (b) equals the compliance $\frac{L}{A E}$, in which case the power for the branch equals

$$P = (F^2_x + F^2_y) R = \frac{F^2 L}{A E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

the ideal transformer consuming no power. With this condition satisfied, and with the components of the loads and reactions duplicated by suitable currents, each resultant branch current F must simulate the corresponding branch force F. In reaching this conclusion it is unnecessary to write out the structural and electrical equations which govern the values of the redundants, for these sets of equations must be identical, by the principles of least work and least power. The latter principle does not apply to the x- and y-network separately, as stated by Mr. Bush and M. Hetenyi, but only to both networks combined.[5]

[5] "Handbook of Experimental Stress Analysis," by M. Hetenyi, John Wiley & Sons, New York, N. Y., 1950, pp. 807–817.

If it is imagined that the x- and y-deflection of the joints of a typical truss are computed by a conventional use of Castigliano's theorem—that is, by applying a load in the desired direction to each joint, writing the work of deformation in terms of the literal (nonnumerical) value of this load, and then applying Eq. 2, after which the load is given its actual value (which may be zero). The corresponding computation for the analog would be similar, but with applied x- or y-current substituted for the load, and with deflection replaced by voltage. Hence, the x- and y-components of deflection at any joint must be simulated by the corresponding voltages, measured below a datum corresponding to points of zero deflection, in the x- and y-networks.

This leads to two interesting conclusions. First, it is unnecessary to maintain at predetermined values the currents corresponding to reaction components, whether the reactions are redundant or not; if the electrical joints at which the corresponding deflection component is zero are grounded, the currents leaving these joints must correspond to the appropriate components of reaction force applied to the truss by its supports. Only the currents corresponding to load components, therefore, must be held at predetermined values.

Second, considering the interpretation of the voltage drops in Fig. 1 (b); the drops across the resistors in the direction BH are $\frac{F_x L}{A E}$ and $\frac{F_y L}{A E}$, which corres-

pond to the respective components of deflection associated with branch elongation. Since the total drops from joints B to H in each electrical branch must equal the corresponding components of total branch deflection, the transformer voltage drops must equal the approximate components of branch deflection caused by rotation. The rotational deflections are ordinarily of the same order of magnitude as the deflections caused by elongation.

In the special case in which a truss branch is parallel to the x-axis, the corresponding transformer and y-branch in the analog are omitted, the x-branch being shorted past its transformer terminals. This assures that the analog of the y-component of branch force is zero. A similar arrangement is used if the branch is parallel to the y-axis.

Considering the truss of Fig. 2(a), which is indeterminate to the second degree, F_{1-5} and F_{3-5} are suitable choices for the redundants. In the analog, Fig. 2(b), the horizontal branches appear only in the x-network, and the vertical branches only in the y-network. The inclined branches appear in both networks, the appropriate x- and y-branches being connected by transformers, the separate windings of which are represented symbolically by circles, and the mutual core by a dotted line, in a manner similar to that used by Mr. Bush. For branches 1–5 and 2–4 the value of N_x/N_y is -1, and for branches 2–6 and 3–5 it is $+1$, by use of Eq. 4. A resistance (represented by a rectangle) corresponding to branch compliance is placed in each electrical branch.

To simulate the constraints imposed on the truss by its supports, joint 6 is

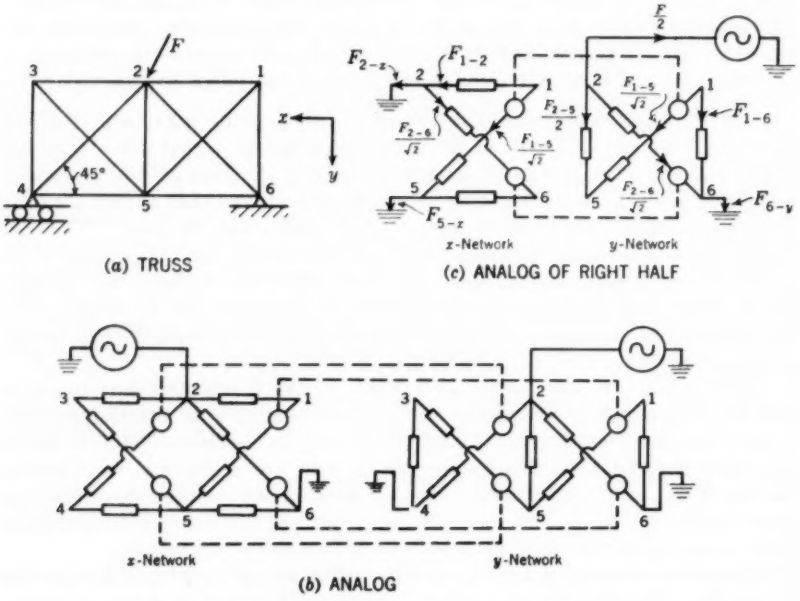


FIG. 2.—TRUSS, INDETERMINATE TO THE SECOND DEGREE

grounded in the x- and y-networks, and joint 4 in the y-network. Appropriate alternating currents, at the same frequency and phase, are withdrawn from joint 2 to simulate the load components.

In the special case in which the load F is in the y-direction the analog can be simplified by the considerations of symmetry. Joints 2 and 5 must have the same x-deflection, so that a vertical plane can be passed through the middle of the load and member 2–5. The datum for the measurement of the x-deflection is conveniently shifted to the plane of symmetry, while the datum for the y-deflection is at joint 6. Under these conditions the analog of only the right half of the truss need be considered, as shown in Fig. 2(c). The resistance in branch 2–5 must equal twice the associated compliance, while the current equals half the corresponding branch force, since the member was cut in half by the plane of symmetry.

The validity of the analog of Fig. 2(c) can be checked by writing the equations of current-continuity and the expression for power, and noting whether they correspond (respectively) to the equations of equilibrium and twice the expression for work in the corresponding half-structure. Each current in the x-circuit is labeled to correspond with the x-component of the prototype branch force, assumed positive in the branch direction of current flow, and similarly for the y-circuit. The transformer turns ratios have been expressly selected to allow this designation. The ground currents are labeled as shown, and are analogous to reaction forces applied to the half-structure. Setting to zero the total current leaving each joint in turn,

Joint	x-circuit	y-circuit
1.....	$F_{1-2} + \frac{F_{1-5}}{\sqrt{2}} = 0$	..... $\frac{F_{1-5}}{\sqrt{2}} + F_{1-6} = 0$
2.....	$-F_{1-2} - \frac{F_{2-6}}{\sqrt{2}} + F_{2x} = 0$	..... $\frac{F_{2-5}}{2} + \frac{F_{2-6}}{\sqrt{2}} + \frac{F}{2} = 0$
5.....	$-\frac{F_{1-5}}{\sqrt{2}} - F_{5-6} + F_{5x} = 0$	..... $-\frac{F_{2-5}}{2} - \frac{F_{1-5}}{\sqrt{2}} = 0$
6.....	$F_{5-6} + \frac{F_{2-6}}{\sqrt{2}} = 0$	..... $-\frac{F_{2-6}}{\sqrt{2}} - F_{1-6} + F_{6y} = 0$

With the branch forces assumed tensile, and with the symmetry of the structure taken into account, these expressions are identically the summations of x- and y-components of force applied to each joint in the truss, so that the requirements of current-continuity and equilibrium are identical.

For the case in which all the truss branch compliances are equal, the power loss for Fig. 2(c) is

$$P = R\,(F^2_{1-2} + F^2_{2-5}/2 + F^2_{5-6} + F^2_{1-6} + F^2_{1-5} + F^2_{2-6}) \ldots\ldots (7)$$

in which the branch resistance R (or $2R$, for branch 2–5) is set equal to the branch compliance $L/(A\,E)$. The work of deformation for the half-structure is half this quantity, so that the analogy is valid. Both the analog and the prototype can be solved analytically by substituting for the forces in Eq. 7

their values in terms of the redundant F_{1-5}, as obtained from the equations of current-continuity or equilibrium, and then setting to zero the partial derivative of the resulting expression with respect to F_{1-5}. The latter then has the value $F\sqrt{2/6}$, from which all other forces, deflection components, currents, and voltages can be determined. This necessitates the evaluation of the transformer voltage drops.

Scale Factor.—The scale factor is the ratio of any quantity in the prototype to its corresponding quantity in the analog. The scale factor relating force to current is S_F, and can be chosen arbitrarily. The scale factor S_C relates compliance (distance/force) to resistance, and also can be chosen arbitrarily. Relating deflection (which is dimensionally equal to force times compliance) to voltage (which is dimensionally equal to current times resistance) is the scale factor S_δ. Hence S_δ is the product of S_F and S_C.

Three-Dimensional Case.—The analog of a typical three-dimensional branch BH is shown in Fig. 3. The transformers relate F_x and F_z to F_y, keeping the currents in the same ratios as the respective x-, y-, and z-components of branch force. Alternatively, these transformers could be replaced by transformers relating F_y and F_x to F_z, or F_z and F_y to F_x. In any case, for transformers relating x- and y-currents, y-, and z-currents, and z- and x-currents, respectively, the transformer turns requirements are

$$\frac{N_x}{N_y} = -\frac{m}{l}, \quad \frac{N_y}{N_z} = -\frac{n}{m}, \quad \frac{N_z}{N_x} = -\frac{l}{n} \qquad (8)$$

in which l, m, and n are the direction cosines of branch BH. In the special case in which a branch is perpendicular to one of the coordinate axes, the electrical branch corresponding to this axis is omitted, and only one transformer is needed for the purpose of relating the currents associated with the other two axes. If the branch is parallel to one of the coordinate axes, the electrical branches corresponding to the other axes are omitted, and no transformers are required.

The connection of the various electrical branches to form a complete analog, and the selection of grounds and the withdrawal of load currents, are governed

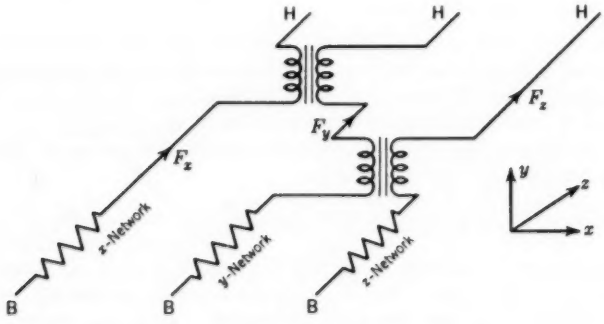


Fig. 3.—Three-Dimensional Truss Member

by the same procedure as in the two-dimensional case. The resistances equal the compliances $L/(A\,E)$, the resistor and transformer voltage drops corresponding to appropriate components of deflection caused by branch elongation and rotation, respectively.

Unification of Branch Resistors.—In both the two-dimensional and three-dimensional cases, the number of resistors can be reduced to one for each structural branch, by choosing a value of resistance which will leave the power dissipation unchanged. By the use of Castigliano's electrical theorem and the principle of least power, this reduction of the number of resistors will leave the analog unchanged with respect to branch currents and joint voltages; the transformer voltage drops will automatically adjust themselves to compensate for the changed resistor voltage drops. For the cases in which it is placed in the x-, y-, and z-branches, respectively, the value of the unified resistance is

$$\frac{R}{l^2},\quad \frac{R}{m^2},\quad \frac{R}{n^2} \qquad (9)$$

For the two-dimensional case, l and m equal $\cos\theta$ and $\sin\theta$, respectively.

Analogs of the Rigid Frame

For simplicity this presentation will be limited to rigid frames and truss-frame combinations in which all loads and members are in one plane. The members may, however, be of arbitrary orientation within the plane, of irregular shape, of varying cross section, and they may have arbitrary loading between the end points. The general scheme of the analogs is to replace force and moment by current, and to arrange the electrical circuit so that the equations of equilibrium and twice the work of deformation are replaced by the equations of current-continuity and power dissipation, respectively. The principles of least power and least work, and Castigliano's theorems for electrical networks and structures then assure currents analogous to force and moment, and voltage drops analogous to deflection and slope.

Straight Member of Constant Cross Section, Loaded at End Points Only.—The typical member BH of length L is shown in Fig. 4(*a*), the parameters being F, the axial force applied to point B by member BH, and considered positive when tensile; S, the resultant shear applied to B by loads and by adjoining

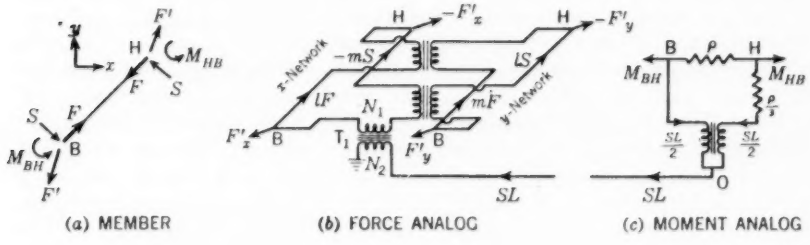


Fig. 4.—Straight Member, Loaded at End Points

members, and considered positive when it tends to rotate BH in the direction of positive slope (which is the direction in which the y-coordinate is 90° removed from the x-coordinate); and M_{BH} and M_{HB}, the resultant moments applied to B and H, respectively, by loads and by adjoining members, and considered positive in the positive direction of slope. The forces at H are equal and opposite to those at B. Denoting by F'_x and the F'_y the components of the resultant force applied to B by the loads and by members other than BH, and noting that the resultant is also the resultant of S and $-F$,

$$F'_x = m\,S - l\,F \quad \ldots\ldots (10a)$$

$$F'_y = -\,l\,S - m\,F \quad \ldots\ldots (10b)$$

Moment summation yields

$$M_{BH} + M_{HB} + S\,L = 0 \quad \ldots\ldots (11)$$

The x- and y-networks of an electrical analog for the forces are shown in Fig. 4(*b*), where the electrical quantities are denoted by the same symbols as the corresponding structural quantities. Current leaving a joint is analogous to a force component applied to the corresponding structural joint. Ideal transformers are used to fix the ratios of the currents, the required turns ratios being, for axial force, $\frac{N_x}{N_y} = -\frac{m}{l}$ and for shear force, $\frac{N_x}{N_y} = \frac{l}{m}$.

The equations of current-continuity at joints B and H simulate Eqs. 10 of force summation. If the tensile and shear work is appreciable (unusual in the case of rigid frames), it can be simulated by inserting suitable resistors in the branches. By analogy with the tensile case, the shear resistance may be written $\frac{\alpha\,L}{A\,G}$, in which G is the shear modulus, and α is the ratio of shear stress at the centroidal plane to the average shear stress S/A.[6] It can be shown by Castigliano's theorems that the branch voltage drops simulate the corresponding components of total branch deflection. The resistor voltage drops in the tension and shear branches simulate deflection components caused by direct tension and direct shear, respectively.

[6] "Strength of Materials, Part 1," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 1940, p. 170.

In the special case in which the member is parallel to the x-axis, the currents analogous to x-shear and y-tension must be zero, so that the corresponding electrical branches are opened, no transformers then being required; and conversely for the case in which the member is parallel to the y-axis.

The moment relationships can be simulated in various ways, but a preferred form of the analog is indicated in Fig. 4(*c*). with transformer turns ratio equal to -1. The $S\,L$-current can be obtained by the indicated transformer tie-in with the x-shear branch, the turns ratio being

$$\frac{N_1}{N_2} = \frac{L}{m} \quad \ldots\ldots (12)$$

in which the units for L are arbitrary. Currents leaving joints B and H simu-

late moments applied to the corresponding structural joints. The equation of current-continuity is identical to Eq. 11 for moment equilibrium, while the power dissipation is

$$P = \rho \left[M^2_{BH} + M_{BH} S L + \frac{(S L)^2}{3} \right] \quad \ldots\ldots\ldots\ldots (13)$$

in which ρ is resistance analogous to bending compliance. The work of bending a straight beam is

$$W_b = \int_0^L \frac{M^2_s \, ds}{2 E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which M_s is the moment at distance s from joint B, and I is the moment of inertia (here assumed constant) of the cross-sectional area. Substituting for M_s its value in terms of M_{BH}, s and S, and integrating, it is found that twice the work of deformation equals the power dissipation if

$$\rho = \frac{L}{E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

By Castigliano's theorems, it can be shown that the voltage drop from B to H in Fig. 4(*c*) represents the change of slope between B and H caused by bending. Also, the voltage drop from B to O represents $\frac{1}{L}$ times the bending deflection of H from the line tangential to the beam at B, in the direction of positive shear; and correspondingly for the voltage drop from H to O. The analog simulates conditions at the joints only, leaving the conditions along the structural branch to be computed separately.

To construct the analog for an entire rigid frame, points B and H for all branches are interconnected according to the configuration of the structure, a separate network being required for x-forces, y-forces, and moments. Electrical joints corresponding to points of zero x-deflection, y-deflection, or slope are grounded, and alternating currents of the same frequency and phase are withdrawn to simulate applied force components and moments. In many cases the force networks may be omitted, as will be shown.

Some Special Members.—For simplicity, it will be assumed that in members other than the simple type previously described the work associated with direct shear and tension is negligible, so that only the moment analog need be considered. The general method of approach is to simulate the conditions of moment equilibrium and the bending work of deformation. By Castigliano's theorems, voltage drop between the joints which represent the ends of the member is analogous to change of slope.

Fig. 5(*a*) shows a beam subjected to uniformly spaced loads Q. The analog is shown in Fig. 5(*b*), in which q is the number of equal spaces between the loads. If the load Q exists at B, it may be added to S. The work associated with Q alone is not correctly simulated, but this work has a derivative only with respect to Q, and is therefore of no interest in the differentiations associated with Castigliano's theorem and the principle of least power. The analog may be reduced to the case of a uniformly distributed load Q', by letting the number of loads

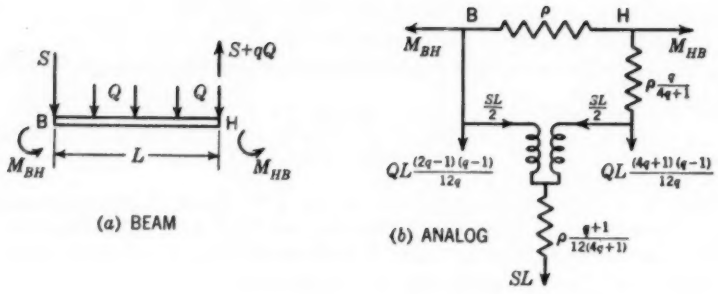


FIG. 5.—BEAM UNDER CONCENTRATED LOADS

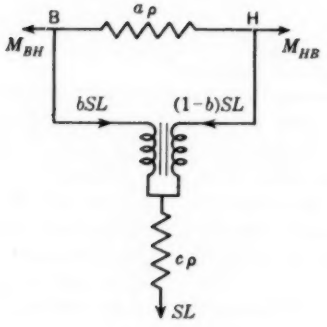


FIG. 6.—ANALOG OF A STRAIGHT MEMBER WITH VARYING MOMENT OF INERTIA

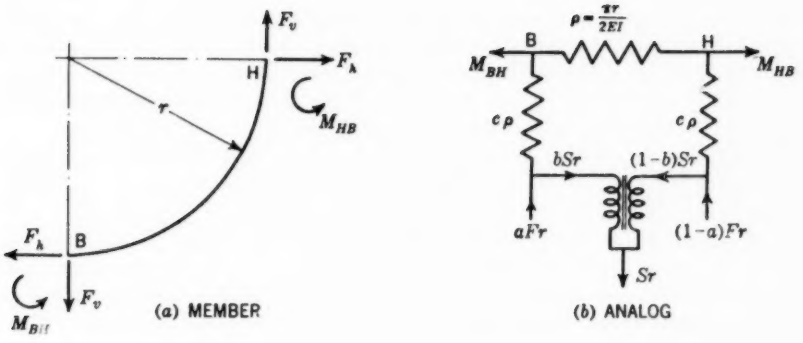


FIG. 7.—QUADRANT MEMBER

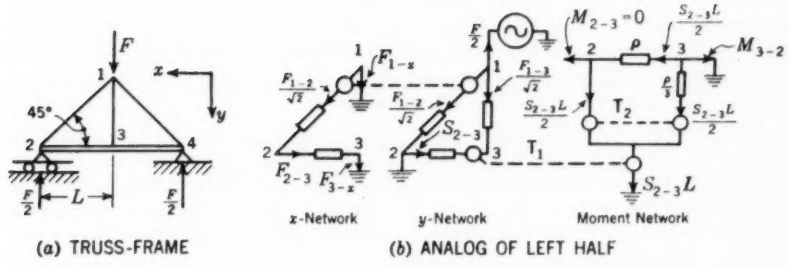


FIG. 8.—TRUSS-FRAME COMBINATION

equal infinity.

Fig. 6 shows the analog of a beam loaded at the end points, and with a moment of inertia I varying linearly from I_B at B to I_H at H. The resistances are always positive, irrespective of the relative magnitudes of I_B and I_H. In Fig. 6

$$a = \log_e i \quad \ldots\ldots (16a)$$

$$b = \frac{1}{\log_e i} - \frac{1}{i - 1} \quad \ldots\ldots (16b)$$

$$c = \tfrac{1}{2} + \frac{1}{i - 1} - \frac{1}{\log_e i} \quad \ldots\ldots (16c)$$

and

$$\rho = \frac{L}{E\,(I_H - I_B)} \quad \ldots\ldots (16d)$$

in which $i = I_H/I_B$.

A quadrant member is shown in Fig. 7(*a*), and its analog in Fig. 7(*b*), in which F_b and F_r are the unknown end forces exerted by adjacent members and $a = 0.3634$, $b = 0.6366$, and $c = 0.1880$. The sign of F_b must be reversed if the quadrant curves are positive in a clockwise direction from B to H. The simulation of twice the work of deformation is not perfect, the excess power P_{ex} being

$$P_{ex} = 0.0063\,\rho\,[(F_r\,r)^2 + (F_b\,r)^2] \quad \ldots\ldots (17)$$

This excess power can usually be neglected, for it is of the order of magnitude of the direct shear and tension work, which is ordinarily ignored in rigid frame problems.

Illustrative Examples.—Fig. 8(*a*) represents a truss-frame combination in which the lower chord has an appreciable resistance to shear and bending, whereas the other members are slender, and thus act almost solely in tension and compression. The structure can also serve as a simple example of a rigidly connected truss in which some of the members are not slender, so that secondary stresses (associated with bending) are significant.

The analog is based only on the left half of the structure. The force networks, Fig. 8(*b*), are based on Fig. 4(*b*), the shear branches for member 1–2 and member 1–3 being omitted. The resistance in branch 1–3 must be doubled to account for the fact that this member is split in half. Joints 1 and 3 have the same x-deflection, by symmetry, and are chosen as the x-deflection datum, while the y-deflection datum is placed at joint 2. The applied current is $F/2$, by symmetry.

The moment network is based on Fig. 4(*c*), and is open at joint 2 to indicate zero moment, and grounded at joint 3 to indicate the slope datum. The $S_{2-3}\,L$-current is obtained by transformer tie-in with the shear branch for member 2–3. The unit of length may be chosen arbitrarily so that the turns ratio of transformer T_1 has any convenient value.

The moment network can be simplified by the elimination of transformer T_2, as follows: The total power in the moment network is determined to be $\frac{\rho\,(S_{2-3}\,L)^2}{3}$. This is simulated by passing the $S_{2-3}\,L$-current through a resist-

ance of value $\rho/3$, as shown in Fig. 8(*d*). The disadvantage of this method is that there is no way of measuring the slope at joint 2, which in Fig. 8(*c*) is simply the voltage of joint 2 below ground.

The validity of the analog of Figs. 8(*b*) and 8(*c*) is checked by comparing the equations of current-continuity and the expression for power with the equations of equilibrium and the expression for twice the work of deformation. All currents are labeled in accordance with Fig. 4, account being taken of the orientation of the coordinate axes in Fig. 8(*a*), and of the fact that M_{2-3} is zero. The ground currents are labeled as shown, and represent force and moment reactions acting on the half-structure. The summations of currents leaving the various joints are shown in Table 1.

TABLE 1.—CURRENTS LEAVING STRUCTURAL JOINTS

Joint	x-Network	y-Network	Moment-network
1	$\frac{F_{1-2}}{\sqrt{2}} + F_{1-x} = 0$	$\frac{F_{1-2}}{\sqrt{2}} + \frac{F_{1-3}}{2} + \frac{F}{2} = 0$	
2	$-\frac{F_{1-2}}{\sqrt{2}} - F_{2-1} = 0$	$-\frac{F_{1-2}}{\sqrt{2}} - S_{2-1} + F_{2y} = 0$	
3	$F_{2-1} + F_{3-x} = 0$	$-\frac{F_{1-3}}{2} + S_{2-3} = 0$	$\frac{2\,S_{2-3}\,L}{2} + M_{1-2} = 0$

With the sign conventions of Fig. 4(*a*), and with the symmetry of the structure considered, the equations of equilibrium are identical to the expressions of Table 1.

The power loss in Fig. 8(*b*) is

$$P = F^2_{1-2}\,R_{1-2} + \frac{F^2_{1-3}\,R_{1-3}}{2} + F^2_{2-3}\,R_{2-3} + S^2_{2-3}\,\frac{\alpha\,E\,R_{2-3}}{G} \quad \ldots\ldots (18)$$

in which R is the branch resistance, except in the case of branches 1–3 and 2–3 in the y-network, in which the resistance is $2\,R_{1-3}$ and $\frac{\alpha\,E\,R_{2-3}}{G}$, respectively. If each R is set equal to the corresponding tensile compliance, then $\frac{\alpha\,E\,R_{2-3}}{G}$ is shear compliance, and Eq. 18 represents twice the tensile and shear work of the prototype half-structure.

The power dissipated in Fig. 8(*c*) is

$$P = \frac{(S_{2-3}\,L)^2\,\rho}{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Eq. 19 equals twice the bending work of the half-structure. All requirements for the validity of the analog are thus met.

Both the prototype and the analog can be solved by expressing the unknown forces and moments in terms of S_{2-3} (which is redundant) by means of the equations of current-continuity or equilibrium, and then setting to zero the partial derivative of the sum of tensile, shear, and bending work with respect to S_{2-3}. For the specific case in which $A_{1-2} = A_{1-4} = A_{1-3}\sqrt{2}$; A_{2-3} and A_{3-4} are large enough so that the work of direct shear and tension in these members is negligi-

ble; and $I_{2-3} = I_{3-4} = \frac{A_{1-3} L^2}{12}$, the value of S_{2-3} is found to equal $-F/8$. With this value known, all other forces, moments, deflections, slopes, currents, and voltages can be evaluated.

Scale Factors.—In the force-networks the scale factors S_F and S_C are arbitrary. The scale factor S_δ equals $S_F S_C$.

In the moment-network, the scale factor S_M, relating moment to current, can be made the same as S_F, in which case it can be shown that S_B, the scale factor relating bending compliance $\left(\frac{\text{dimensions} = 1}{\text{force} \times \text{distance}}\right)$ to resistance, is the same as S_C, and that S_θ, the scale factor relating slope (in radians) to voltage, equals S_δ. The power dissipation and the equations of current-continuity will be unchanged, however, if all currents are multiplied by any arbitrary factor f and the resistances by $1/f^2$, the voltages then being $1/f$ as large as before. The scale factors in the moment-network are then

$$S_M = \frac{S_F}{f} \quad \ldots\ldots (20a)$$

$$S_B = S_C f^2 \quad \ldots\ldots (20b)$$

$$S_\theta = S_\delta f \quad \ldots\ldots (20c)$$

As a second example, Fig. 9(*a*) shows a rigid frame, loaded by a force F, in which the work of direct tension and shear is negligible, only the bending work being of interest. Since no force networks are used, S_{1-2} and S_{2-3} must be determined in terms of the load and the redundants, and the corresponding shear currents withdrawn from the bending network. By equilibrium considerations, it can be found that there is only one redundant, conveniently selected as the reaction F_1. In terms of F and F_1

$$S_{1-2} = \frac{F_1}{\sqrt{2}} \quad \ldots\ldots (21a)$$

and

$$S_{2-3} = F_1 - F \quad \ldots\ldots (21b)$$

The analog, shown in Fig. 9(*b*), can be developed as follows: The networks for member 1–2 and member 2–3 are first drawn according to Fig. 4(*c*), and are interconnected according to the configuration of the structure. Joint 3 is grounded to establish the slope datum, and joint 1 is opened to indicate zero moment. The shear currents must be interrelated according to Eqs. 21, without changing the number of redundants (in this case, one). This can be accomplished by selecting one shear current as redundant, then developing the other currents by means of transformers and generators, as necessary. The selected redundant may be considered as $S_{2-3} L$. This is separated into its component parts by use of generator G_1, the shear current for member 1–2 ($F_1 L$) being established by means of transformer T_1. For uniform cross section throughout the structure

$$F_1 = \frac{5 F}{(14 + 2\sqrt{2})} \quad \ldots\ldots (22)$$

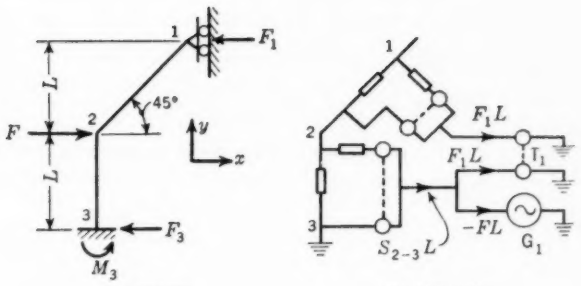


FIG. 9.—RIGID FRAME

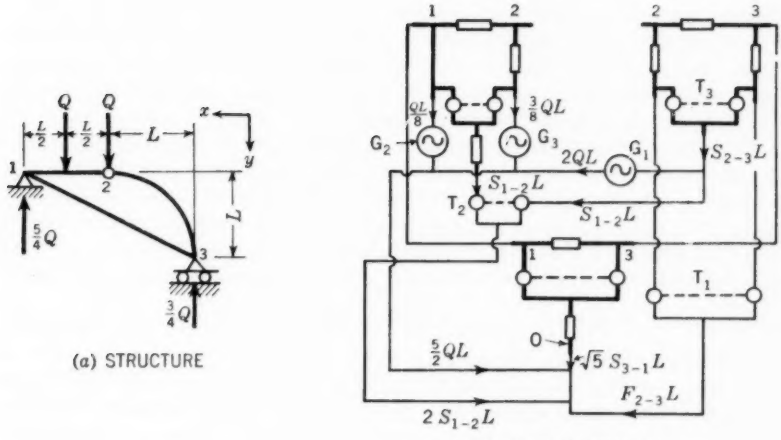


FIG. 10.—IRREGULAR STRUCTURE

which can be used to check the validity of the entire analog.

Fig. 10(*a*) represents an irregular structure, in which the work of direct tension and shear is considered insignificant. The joints are rigid except for joint 2, which is hinged. Member 1–2 and member 2–3 have the same constant moment of inertia I, while the moment of inertia for member 1–3 varies linearly from I at joint 1 to $2\,I$ at joint 3. The reactions are determinate, and are as indicated in the figure.

To determine the bending network without using the force networks, it is necessary to find S_{1-2}, S_{3-1}, S_{2-3}, and F_{2-3} (the two latter terms being the shear and tension, respectively, in member 2–3 and joint 2) in terms of the loads and redundants. Applying the equations of force summation to each joint,

$$S_{2-3} = S_{1-2} + 2\,Q \qquad (23a)$$

$$S_{3-1} = -\frac{F_{2-3} + 2\,S_{1-2} + \frac{5}{2}\,Q}{\sqrt{5}} \qquad (23b)$$

in which S_{1-2} and F_{2-3} are selected as redundants. Applying moment summation to each member, it is found that there are no other redundants.

The bending analog, Fig. 10(*b*), may be developed as follows: The networks for the separate members are drawn according to Figs. 5, 6, and 7, and are interconnected according to the configuration of the structure, the connection between member 1–2 and member 2–3 being omitted to indicate zero moment at joint 2. Currents $S_{2-3}\,L$ and $F_{2-3}\,L$ are arbitrarily selected as the two redundants, it being necessary to develop the remaining shear currents, $S_{1-2}\,L$ and $\sqrt{5}\,S_{3-1}\,L$, by use of transformers and generators.

Current $S_{2-3}\,L$ is separated into its component parts by means of generator G_1. The part designated as $S_{1-2}\,L$ is then used to set up the shear current for member 1–2, with the aid of transformer T_2. To set up the shear current for member 3–1, the negative of the sum of the following three currents is withdrawn from network 3–1: $F_{2-3}\,L$, obtained from branch 2–3 as the combined current passing through transformer T_1; $2\,S_{1-2}\,L$, obtained as the combined current passing transformer T_2; and $\frac{5}{2}\,Q\,L$, obtained as the sum of the currents supplied by generators G_1, G_2, and G_3.

No choice of slope datum need be made, since the analog deals with slope differences. The actual slope at any point, for example, θ_1, at joint 1, can be determined by a method which parallels conventional structural technique: That is, the y-deflection of member 1–3 is expressed thus:

$$\theta = \frac{2}{\sqrt{5}}\,\delta_{1-3} + \theta_1\,2\,L \qquad (24)$$

in which δ_{1-3} is the deflection of joint 3 with respect to joint 1, measured from the tangential line through joint 1 in the direction of positive shear. By Castigliano's theorem, the voltage drop from 1 to O of network 1–3 is $\frac{\delta_{1-3}}{L\sqrt{5}}$, so that θ_1 is readily evaluated.

The values of the redundants selected for the structure are found by analysis to be:

	F_{2-3}	S_{1-2}
Structure	$-1.511\ Q$	$-0.804\ Q$
Analog	$-1.489\ Q$	$-0.808\ Q$

The slight difference in the values is caused by the fact that the power dissipation in the analog is in excess, as explained for Fig. 7. With these values known, a complete check of the entire analog is facilitated.

Experimental Equipment, Procedure, and Results

Fig. 11 shows a working model, the four essential parts of which are:

(1) Resistors. These are in the form of thirty calibrated rheostats, with the actuating knobs visible on the front panel. There are three types, with maximum resistance values of 100 ohms, 300 ohms, and 1,000 ohms, respectively. The resistances may be set to within 1% of the maximum value, and are used both in the analog and in conjunction with voltage sources to be described subsequently as a means of controlling load currents. Connections to the rheostats are made by use of the horizontally disposed screw-type terminal strips, each terminal being numbered to agree with the rheostat to which it is connected.

(2) Supply Transformers. These are of a conventional type, with 115-volt, 60-cycles-per-sec input, and with a plurality of independent 6-volt center-tapped secondaries, the latter being used as voltage sources or generators. Connection to the secondaries, of which there are six, is made by means of the last two vertically disposed terminal strips at the right of the photograph. The minimum resistance placed in series with each voltage source should be large enough to limit the current to a safe value in case of a short circuit in the analog.

(3) Computing Transformers. These are used in the analog, and have a nickel-iron core of about $\frac{1}{4}$ sq in. cross-sectional area, and two windings of five hundred turns of 24-gage wire. One of the windings is divided into five independent parts having the following percentages of the total turns: 1%, 3%, 9%, 27%, and 60%. By connecting some or all of these in series, bucking or aiding, it is possible to vary the turns ratio (or its reciprocal) in steps of 1% from zero to 100%. There are twelve such transformers, each having its windings connected to one of the vertically disposed terminal strips, by means of which the turns ratio is selected and connection is made to the other elements of the analog.

(4) Measuring Circuit. This consists of the alternating current microammeter shown in Fig. 11, and an accurate rheostat with a dial arrangement which permits precise setting over the full range of ten turns. These elements can be used to measure voltage without drawing current from the analog, with the one necessity being to find the rheostat setting at which no current (in practice, minimum current) flows through the meter. For measuring negative voltage, the connections to the measured voltage are merely interchanged. Current is measured by noting the voltage drop across some known resistance.

Transformer Imperfections.—The accuracy of the instrument is limited

chiefly by the departure of the computing transformers from ideality. At a frequency of 60 cycles per sec the important transformer imperfections are as follows:

(1) Equivalent Shunt Inductance. This arises from the fact that the transformer draws a small magnetizing current. Its effect can be practically eliminated by shunting a suitable fixed capacitance across the fixed winding. Such correction was not found necessary, however.

(2) Series Inductance. This is caused by leakage flux, and can be remedied by inserting a suitable fixed capacitor in series with the fixed winding, but this also was not considered necessary.

(3) Equivalent Shunt Resistance. This is associated with eddy current and hysteresis losses in the core, and can be dealt with as follows: As each transformer drop is measured, the resulting current drawn by the equivalent shunt resistance, about 7,500 ohms, is estimated. If this current is appreciable, in comparison to the analog currents, there is imposed on the transformer a compensating value of in-phase ampere turns of the correct magnitude and polarity just to sustain the core losses. This is preferably done by passing a suitable current through a special winding of few turns, so that the induced electromotive force will be small enough not to complicate the adjustment of the inserted current. The whole operation can be done very rapidly by means of prepared charts relating transformer voltage drops to values of resistance which, when placed in series with a selected voltage source, will pass the required compensating current. This correction also was not used.

(4) Series Resistance. Series resistance is the resistance of each transformer winding (2 ohms for the fixed winding) and is compensated for by deducting it from a convenient analog resistance with which it is in series, the latter usually being much larger. For a given winding, the resistance to be deducted can also be referred to any analog resistance whose current is held in a fixed ratio with the winding current, by means of the following formula, which will leave the power unchanged:

$$R_r = R_w \frac{C^2_w}{C^2_r} \qquad (25)$$

in which R_r and R_w are the referred and winding resistances, respectively, and C_r and C_w the referred and winding currents, respectively. This correction was used throughout the investigation.

Experimental Procedure.—The steps in setting a problem into the equipment are as follows:

1. The circuit diagram is drawn according to the various examples previously cited. Trusses and truss-frame combinations follow the discussions given in connection with Figs. 2 and 8. For rigid frames in which only the bending work is of interest, only the moment analog need be drawn. The forces which influence the bending work are determined separately in terms of the loads and redundants, the corresponding currents then being withdrawn from the analog, as in Figs. 9 and 10.

2. A scale factor is chosen between the structural compliances $\left(\frac{L}{A\,E'}, \quad \frac{L}{E\,I}\right)$

and the corresponding electrical resistances. The latter is small in comparison with the equivalent shunt resistances of the transformers, and large compared to the winding resistances, the optimum compromise probably being reached with resistances of from 50 ohms to 300 ohms. The computing transformers are connected to yield the required turns ratios. After that, they are wired to the selected rheostats, which have been set to the proper values. The ground or datum points are then connected together.

3. Load currents are supplied by the voltage sources in series with resistances, the voltage drop across the resistances serving as an indication of current. The scale factor between force and moment and the corresponding currents is a matter of convenience.

4. The branch currents and joint voltages are measured. The former represent force or moment, which are readily converted to stress, and the latter represent slope or deflection. If the structure is seen to be of unsatisfactory design, the cross section of selected members can be altered by changing the setting of the associated rheostats. This change usually necessitates a readjustment of the load currents. Measurements are then made as before, and the whole process is repeated until the design is satisfactory.

5. A rapid numerical check can be made by noting whether the currents leaving each joint and the voltage drops around each current path sum to zero. This, however, does not reveal errors in the circuit diagram. A more thorough check can be conducted in purely structural terms, by noting whether the results satisfy the requirements of equilibrium, and whether the deflections and slopes are mutually compatible.

Experimental Results.—The structures of Figs. 2, 8, 9, and 10 were solved experimentally. The error is expressed as a percentage of the maximum values encountered. This manner of expressing error is considered more indicative of the overall accuracy than if the error is expressed as a percentage of each individual quantity. The latter can be much larger than the former, for members that are relatively lightly loaded. However, the error is of far less importance for these members than for the heavily loaded members.

The computing transformers were uncompensated, except that the winding resistances were subtracted from analog resistances where possible.

The units used were as follows: Resistance in ohms, voltage in divisions of the ten-turn rheostat, and current in divisions per ohm. In the experiments, one thousand divisions represented 6 volts.

Fig. 2(*c*). The branch resistance R was 100 ohms (200 ohms for branch 2–5), and F was 6.58. The joint voltages (from which the branch currents are computed) were measured, the greatest error compared to theoretical values being 2% of $F\,R$.

Fig. 8. The tensile compliance was represented by 100 ohms. For L equal to unity, the bending compliance resistor ρ would have been 1,200 ohms, which was considered too high compared to the transformer equivalent shunt resistance of 7,500 ohms. Hence L was set at 2, thus reducing ρ to 300 ohms, transformer T_1 then having a 2:1 turns ratio. The value of F was 7.94. The various currents were measured, the greatest error being 1.5% of F.

Fig. 9. The bending compliance resistor was 300 ohms for member 1–2

and $F\,L$ was 8.40. The currents were measured and were found to be accurate to within 1/2% of $F\,L$.

Fig. 10. The resistance simulating bending compliance was 300 ohms for member 3–1 and $Q\,L$ was 4.545. The voltages and currents were measured, the latter having the larger error—2.5% of $Q\,L$ as referred to the theoretical solution for the electrical analog. The small difference between the structure and its analog need not be charged against the latter, since it is caused by an excess of power comparable to the work caused by direct shear and tension, which is customarily ignored in rigid-frame computations. It was not possible to absorb the winding resistances of transformers T_1 and T_3 in any analog resistances, which may account for the somewhat larger error observed in this analog.

Conclusions

For two-dimensional and three-dimensional trusses, the analog developed by Mr. Bush has been extended to include the simulation of indeterminate reactions and of deflections caused by branch rotation.

For two-dimensional rigid frames there has been presented a new analog in which slope and deflection need not be simulated explicitly, as in Dr. Bush's

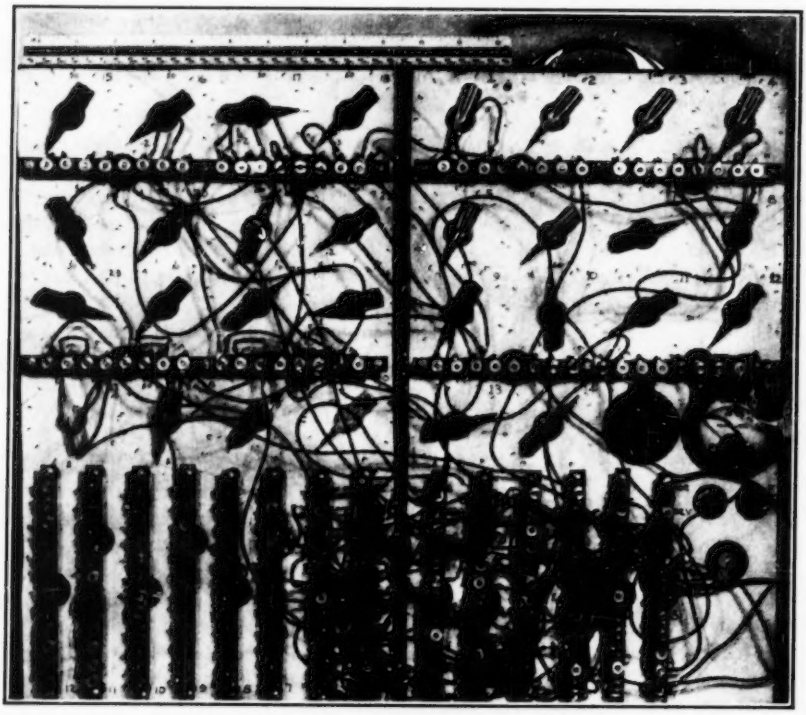


Fig. 11.—Measuring Instrument

analog, but are obtained automatically with a simulation based on force, moment, and elastic compliance. Further, there is no limitation on the orientation or configuration of the members.

For trusses, the circuit diagram can be constructed on a purely pictorial basis. For rigid frames, the analog achieves its maximum economy when it is only partly pictorial, in which case it is necessary to solve the equations of equilibrium. It is unnecessary to derive the equations governing the mutual compatibility of the slopes and deflections, such as appear in explicit form in the method of slope deflection, in implicit form in the method of least work, and in iterative numerical form in the method of moment distribution. For irregular members, however, it is necessary to derive special analogs, as shown in Figs. 5, 6, and 7. Such analogs may be tabulated and used wherever applicable.

For the experimental solution of practical problems, an instrument such as that of Fig. 11, but large enough to accommodate structures of many members, may be necessary. This instrument would contain no component of unusual cost or difficulty of manufacture, or of limited life, such as vacuum tubes. The expected error would be of the order of 2% of the maximum values encountered in each problem. Should the error increase substantially for structures of many members, the computing transformers, which are the chief source of error, must be compensated.

With such an instrument in the hands of an experienced operator, it is estimated that the total experimental solution, including the construction of the circuit diagram, the making of the connections and setting of the values, and the measuring of the unknowns and converting them into structural quantities, will require a small fraction of the time required for analytical solution, for structures of at least moderate complexity and indeterminacy. The saving in time will be particularly great when an optimum design is sought, so that different choices of dimensions are to be explored. In such cases the successive solutions are obtained in a small fraction of the time required for the initial solution, in contrast with conventional analytical methods, where much of the solving procedure must be repeated for each choice of dimensions.

Acknowledgment

This paper was based on a thesis which has been accepted by the faculty of the Graduate Division of the College of Engineering of New York University, in New York, N. Y., in partial fulfilment of the requirements for the degree of Doctor of Engineering Science.

APPENDIX. NOTATION

The following symbols, adopted for use in this paper and for the guidance of discussers, conform essentially with American Standard Letter Symbols for Structural Analysis (ASA-Z10.8–1949), prepared by a committee of the American Standards Association, with Society representation, and approved by the Association in 1949:

A = cross-sectional area of a member;
B = end of a typical member;
C = supplied electrical current:
C_r = referred currents;
C_w = windings current;
E = modulus of elasticity;
F = axial force, current analogous to force:
F_x, F_y = components of the axial force;
F'_x, F'_y = components of the resultant of all forces on the end of a member;
F_h, F_v = horizontal and vertical components of a force;
f = scale factor coefficient;
G = shear modulus;
$G_{1,2,\ldots}$ = generators in electrical network;
H = end of a typical member;
I = moment of inertia of a section;
L = length of a member;
l, m, and n = direction cosines of a member;
M = applied bending moment;
N = transformer turns ratio;
O = point in an electrical network;
P = electric power dissipation:
P_{ex} = excess power in an electrical network;
Q = concentrated loads;
Q' = uniformly distributed load;
q = number of spaces between concentrated loads;
R = branch resistance:
R_r = referred resistance;
R_w = windings resistance;
r = radius of curvature of curved member;
S = shear in member, current analogous to shear;
S_B = scale factor relating bending compliance to resistance;
S_C = scale factor relating compliance to resistance;
S_F = scale factor relating force to current;
S_M = scale factor relating moment to current;
S_δ = scale factor relating deflection to voltage;
S_θ = scale factor relating slope to voltage;
T_1, T_2 = transformers in electrical network;
V = voltage in electrical network;
W_b = work of bending;
W_d = work of deformation;
x, y, and z = rectangular coordinates;
α = ratio of maximum shear stress to average shear stress;
δ = elastic deflection;
θ = elastic slope of a beam; and
ρ = resistance analogous to bending compliance.

PROCEEDINGS-SEPARATES

VOLUME 79 (1953)

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MAY: 189(HY), 190(HY), 191(CP) & (AT), 192(SM) 193(HY), D-129(PO), D-138(CP), D-145(ST).

JUNE: 194(CP) & (AT), 195(SM), 196(CP) & (AT), 197(HY), 198(ST), 199(EM), D-134(HY), D-141(HY).

JULY:[a] 200(SM),[b] 201(ST),[b] 202(EM),[b] 203(SA),[b] 204(AT),[b] 205(EM),[b] 206(ST),[b] 207(SA),[b] 208(SA),[b] 209(ST),[b] 210(SU),[b] 211(EM),[b] 212(SU),[b] 213(IR),[b] 214(HW),[b] 215(SM),[b] 216(ST),[b] 217(ST),[b] 218(ST),[b] 219(ST),[b] 220(SM),[b] 221(HW),[b] 222(SM),[b] 223(EM),[b] 224(EM),[b] 225(EM),[b] 226(CO),[b] 227(SM),[b] 228(SM),[b] 229(IR).[b]

AUGUST: 230(HY), 231(SA), 232(SA), 233(AT), 234(HW), 235(HW), 236(HW), 237(AT), 238(WW), 239(SA), 240(IR), 241(AT), 242(IR), 243(ST), 244(ST), 245(ST), 246(ST), 247(SA), 248(SA), 249(ST), 250(EM),[c] 251(ST), 252(SA), 253(AT), 254(HY), 255(AT), 256(ST), 257(SA), 258(EM), 259(WW).

SEPTEMBER: 260(AT), 261(EM), 262(SM), 263(ST), 264(WW), 265(ST), 266(ST), 267(SA), 268(CO), 269(CO), 270(CO), 271(SU), 272(SA), 273(PO), 274(HY), 275(WW), 276(HW), 277(SU), 278(SU), 279(SA), 280(IR), 281(EM), 282(SU), 283(SA), 284(SU), 285(CP), 286(EM), 287(EM), 288(SA), 289(CO).

OCTOBER:[d] 290(all Divs), 291(ST),[c] 292(EM),[c] 293(ST),[c] 294(PO),[c] 295(HY),[c] 296(EM),[c] 297(HY),[c] 298(ST),[c] 299(EM),[c] 300(EM),[c] 301(SA),[c] 302(SA),[c] 303(SA),[c] 304(CO),[c] 305(SU),[c] 306(ST),[c] 307(SA),[c] 308(PO),[c] 309(SA),[c] 310(SA),[c] 311(SM),[c] 312(SA),[c] 313(ST),[c] 314(SA),[c] 315(SM),[c] 316(AT),[c] 317(AT),[c] 318(WW),[c] 319(IR),[c] 320(HW).[c]

NOVEMBER: 321(ST), 322(ST), 323(SM), 324(SM), 325(SM), 326(SM), 327(SM), 328(SM), 329(HW), 330(EM), 331(EM), 332(EM), 333(EM), 334(EM), 335(SA), 336(SA), 337(SA), 338(SA), 339(SA), 340(SA), 341(SA), 342(CO), 343(ST), 344(ST), 345(ST), 346(IR), 347(IR), 348(CO), 349(SM), 350(HW), 351(HW), 352(SA), 353(SU), 354(HY), 355(PO), 356(CO), 357(HW), 358(HY).

DECEMBER: 359(AT), 360(SM), 361(HY), 362(HY), 363(SM), 364(HY), 365(HY), 366(HY), 367(SU),[e] 368(WW),[e] 369(IR), 370(AT),[e] 371(SM),[e] 372(CO),[e] 373(ST),[e] 374(EM),[e] 375(EM), 376(EM), 377(SA),[e] 378(PO).[e]

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a. Beginning with "Proceedings-Separate No. 200," published in July, 1953, the papers were printed by the photo-offset method.

b. Presented at the Miami Beach (Fla.) Convention of the Society in June, 1953.

c. Presented at the New York (N.Y.) Convention of the Society in October, 1953.

d. Beginning with "Proceedings-Separate No. 290," published in October, 1953, an automatic distribution of papers was inaugurated, as outlined in "Civil Engineering," June, 1953, page 66.

e. Discussions, grouped by divisions.

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